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Sri Lanka Institute of Information Technology

IT0060 –Essential Mathematics

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Partial Fractions

Rational Expressions

- **What is a Rational Expression?**
  - A rational expression is a fraction where:
  - Numerator and/or denominator are **polynomials**
  - Denominator is **not equal to zero**
- **General Form:**

$$\frac{P(x)}{Q(x)}, Q(x) \neq 0$$

- **Examples:**
  - $\frac{2x}{x+3}$
  - $\frac{x^2-1}{x-1}$

Degree of an Equation

The degree of an equation is the **highest power of the variable** in the equation when it is written in standard polynomial form.

Examples

1.

$$3x + 5 = 0$$

Highest power of x is **1**

Degree = 1 (Linear equation)

2.

$$x^2 - 4x + 7 = 0$$

Highest power of x is **2**

Degree = 2 (Quadratic equation)

3.

$$2x^4 + x^3 - x + 9 = 0$$

Highest power of x is **4**

Degree = **4**

4.

$$5x^3 - 2x^2 + 7x - 1 = 0$$

The highest power of x is **3**.

Degree = 3.

Proper vs Improper Fractions

- **Proper Fraction**
  - Degree of numerator < Degree of denominator

$$\frac{x}{x^2+2}$$

- **Improper Fraction**
  - Degree of numerator ≥ Degree of denominator

$$\frac{x^4+x^2+x}{x^3+x+3}$$

Partial Fractions: Distinct Linear Factors

- What is Partial Fraction Decomposition?
  - It is the process of expressing a rational expression as a sum of simpler fractions.
- Starting with: $\frac{3x + 5}{(x - 3)(2x + 1)}$
  - **Step 1:** Set up the form

$$\frac{(3x+5)}{[(x-3)(2x+1)]} = \frac{A}{(x-3)} + \frac{B}{(2x+1)}$$

  - **Step 2:** Multiply both sides by denominator

$$3x + 5 = A\,(2x + 1) + B\,(x - 3)$$

Partial Fractions: Distinct Linear Factors

- **Step 3:** Choose special values to eliminate variables

  - From: $3x + 5 = A\,(2x + 1) + B\,(x - 3)$

    - Let x = $-\frac{1}{2}$: (eliminates A)

      $\frac{7}{2} = -\frac{7}{2}$ x B

      B = -1

    - Let x =3 : (eliminates B)

      14 = 7A

      A = 2

Result: $\frac{(3x+5)}{[(x-3)(2x+1)]} = \frac{2}{(x-3)} - \frac{1}{(2x+1)}$

Partial Fractions: Repeated Factors

- When the denominator has a repeated factor like $(x - 1)^2$

Example: $$\frac{(3x+1)}{(x-1)^2(x+2)}$$

- **Important:** Include terms for both $(x - 1)$ and $(x - 1)^2$

$$\frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$$

- After multiplying by the denominator:

$$3x + 1 = A(x - 1)(x + 2) + B(x + 2) + C(x - 1)^2$$

Partial Fractions: Repeated Factors

- **Technique: Equating Coefficients**
  - When special values aren't enough, compare coefficients of like terms
  - From: $3x + 1 = (A + C)x^2 + (A + B - 2C)x + (-2A + 2B + C)$
- **Coefficient of x^2:**

$$0 = A + C$$

- **Coefficient of x:**

$$3 = A + B - 2C$$

- **Constant term:**

$$1 = -2A + 2B + C$$

- Solve these equations along with values from special substitutions

Partial Fractions: Quadratic Factors

- When the denominator contains a quadratic expression that cannot be factorized
  - **Example:** $\frac{5x}{[(x^2+x+1)(x-2)]}$
  - **Important:** The numerator over a quadratic can contain x

$$\frac{Ax+B}{x^2+x+1}+\frac{C}{(x-2)}$$

- This is still a proper fraction because the numerator degree (1) is less than the denominator degree (2)

Partial Fractions: Quadratic Factors Contd.

- $5x = (Ax + B)(x - 2) + C(x^2 + x + 1)$

  x = 2: $10 = 7C$ → **$C = 10/7$**

  Coefficient of x^2: $0 = A + C$ → **$A = -10/7$**

  Constant term: $0 = -2B + C$ → **$B = 5/7$**

- Result: $\frac{(-10x+5)}{[7(x^2+x+1)]} + \frac{10}{[7(x-2)]}$

Exercises

1. $\frac{17x-53}{x^2-2x-15}$
2. $\frac{125+4x+9x^2}{(x-1)(x+3)(x+4)}$
3. $\frac{7x^2-17x+38}{(x+6)(x-1)^2}$
4. $\frac{4x^3+16x+7}{(x^2+4)^2}$
5. Express f(x) in partial fractions in terms of *k*.

$$f(x) = \frac{3kx-18}{(x+4)(x-2)}$$ where *k* is a positive constant.